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B.Sc. Part II, Paper IV

(Some Problems) DIFFERENTIAL EQUATION (linear Eqns. with constant coefficient)

Q.1. → Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2 + e^x$.

Soln. Here Auxiliary eqn. is $m^2 - 5m + 6 = 0$

$\Rightarrow m^2 - 2m - 3m + 6 = 0 \Rightarrow m(m-2) - 3(m-2) = 0$

$\Rightarrow (m-2)(m-3) = 0 \quad \therefore m = 2, 3$

$\therefore$ C.F $= Ae^{2x} + Be^{3x}$

And P.I $= \frac{1}{D^2-5D+6}(x^2+e^x) = \frac{1}{D^2-5D+6}x^2 + \frac{1}{D^2-5D+6}e^x$ ——— (1)

Now, $\frac{1}{D^2-5D+6}x^2 = \frac{1}{(D-2)(D-3)}(x^2) = \left[\frac{1}{D-3} - \frac{1}{D-2}\right]x^2$

$= -\left[\frac{1}{3-D} - \frac{1}{2-D}\right]x^2 = -\left[\frac{1}{3(1-\frac{D}{3})} - \frac{1}{2(1-\frac{D}{2})}\right](x^2)$

$= \left[\frac{1}{2}\left(1-\frac{D}{2}\right)^{-1} - \frac{1}{3}\left(1-\frac{D}{3}\right)^{-1}\right](x^2)$

$= \left[\frac{1}{2}\left\{1+\frac{D}{2}+\frac{D^2}{4}+\cdots\right\} - \frac{1}{3}\left\{1+\frac{D}{3}+\frac{D^2}{9}+\cdots\right\}\right](x^2)$

$= \frac{1}{2}\left\{x^2+\frac{1}{2}(2x)+\frac{1}{4}(2)\right\} - \frac{1}{3}\left\{x^2+\frac{1}{3}(2x)+\frac{1}{9}(2)\right\}$

$= (x^2)\left(\frac{1}{2}-\frac{1}{3}\right) + (2x)\left(\frac{1}{4}-\frac{1}{9}\right) + (2)\left(\frac{1}{8}-\frac{1}{27}\right)$

$= (x^2)\times\frac{1}{6} + (2x)\times\frac{5}{36} + 2\times\frac{19}{216} = \frac{x^2}{6} + \frac{5x}{18} + \frac{19}{108}$

Also, $\frac{1}{D^2-5D+6}e^x = \frac{e^x}{1-5+6} = \frac{1}{2}e^x$

Hence, from (1), P.I. $= \frac{x^2}{6} + \frac{5x}{18} + \frac{19}{108} + \frac{1}{2}e^x$.

$\therefore$ The complete solution is

$y = Ae^{2x} + Be^{3x} + \frac{1}{2}e^x + \frac{x^2}{6} + \frac{5x}{18} + \frac{19}{108}$

Ans

V.V.I

Q. 2. → Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 + e^x + \cos 2x$ (2)

Soln:- The auxiliary equation is $m^2 - 4m + 4 = 0$

$\Rightarrow (m-2)^2 = 0 \quad \therefore m = 2, 2$

$\therefore C.F = (C_1 + C_2 x)e^{2x}$ [$\because$ roots are equal]

and $P.I. = \frac{1}{D^2 - 4D + 4}(x^2 + e^x + \cos 2x)$

$= \frac{1}{D^2-4D+4}x^2 + \frac{1}{D^2-4D+4}e^x + \frac{1}{D^2-4D+4}\cos 2x$ ——— (1)

Now, $\frac{1}{D^2-4D+4}x^2 = \frac{1}{(D-2)^2}x^2 = \frac{1}{4\left(1-\frac{D}{2}\right)^2}x^2$

$= \frac{1}{4}\left(1-\frac{D}{2}\right)^{-2}x^2 = \frac{1}{4}\left\{1 + 2\cdot\frac{D}{2} + 3\frac{D^2}{4} + \cdots\right\}x^2$

$= \frac{1}{4}\left\{x^2 + 2x + \frac{3}{4}\times 2\right\} = \frac{1}{4}\left(x^2 + 2x + \frac{3}{2}\right)$

Also $\frac{1}{D^2-4D+4}e^x = \frac{1}{1-\cancel{4}+\cancel{4}}e^x = e^x.$

And $\frac{1}{D^2-4D+4}\cos 2x = \frac{1}{\cancel{-4}-4D+\cancel{4}}\cos 2x$

$= -\frac{1}{4}\cdot\frac{1}{D}\cos 2x = -\frac{1}{4}\frac{\sin 2x}{2} = -\frac{1}{8}\sin 2x$

Hence from (1) $P.I. = \frac{1}{4}\left(x^2+2x+\frac{3}{2}\right) + e^x - \frac{1}{8}\sin 2x$

$\therefore$ The complete solution is

$y = (C_1 + C_2 x)e^{2x} + \frac{1}{4}\left(x^2+2x+\frac{3}{2}\right) + e^x - \frac{1}{8}\sin 2x$

Ans